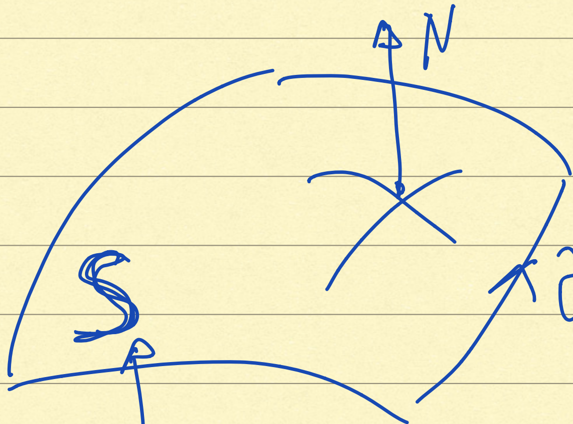
Teorema de Stokes:

$F: \mathbb{R}^3 \to \mathbb{R}^3, C^1$



$\partial S \longrightarrow$ Bordo de S

Superfície limitada orientável

$$\underbrace{\iint_S \text{rot} F \cdot N}_{\text{Fluxo do rotacional}} = \underbrace{\oint_{\partial S} F \cdot d\gamma}_{\text{Trabalho}}$$

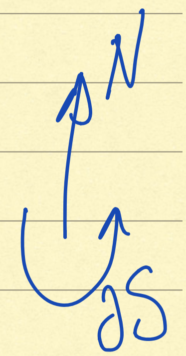
Ex. 3 (F-13):

$$H(x,y,z) = (x^2 - y,\ y^2 - x,\ y^2 - x^2 + z^3)$$

$L: g(t) = (\cos t, \operatorname{sen} t, \cos(2t))$; $0 \le t \le 2\pi$

Se $\boxed{L = \partial S}$, então

$$\int_L H \cdot dg = \iint_S \operatorname{rot} H \cdot N$$



Solução: Construir S tal que

$$\partial S \equiv L.$$

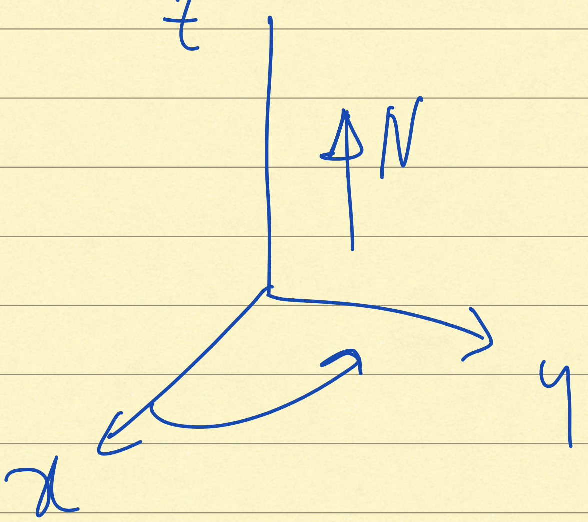
$$g(t): \quad \begin{array}{l} x(t) = \cos t \\ y(t) = \operatorname{sen} t \\ z(t) = \cos(2t) \end{array} \Longrightarrow \underline{2 \text{ equações}}$$

$L: x^2+y^2=1 \ ; \quad z = x^2-y^2$

$\Downarrow$

S (1 equação) limitada

$$\begin{cases} z = x^2 - y^2 \\ x^2+y^2 < 1 \end{cases}$$



$x = \cos t$

$y = \operatorname{sen} t$

$z = \cos(2t)$

$$\int_L H \cdot dg = \iint_S \operatorname{rot} H \cdot N \ , \quad N_z > 0$$

parametrizar S, calcula rot H
etc .... (exercício).

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Ex: 4.6 (F-13)

$S \subset \mathbb{R}^3$, $G : \mathbb{R}^3 \to \mathbb{R}^3$, $N_z > 0$

$$\iint_S \boxed{G} \cdot N = ??? \quad (\text{Stokes})$$

$$\iint_S \boxed{\text{rot } F} \cdot N = \boxed{\oint_{\partial S} F \cdot d\gamma}$$

Solução: Construir $F: \mathbb{R}^3 \to \mathbb{R}^3$ tal que $\boxed{\text{rot}\, F = G}$.

Dado G, resolver esta equação para determinar F.

$$F = (P, Q, R) = ?$$

$$\text{rot}\, F = \text{"}\begin{vmatrix} i & j & k \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ P & Q & R \end{vmatrix}\text{"} = \left(\frac{\partial R}{\partial y} - \frac{\partial Q}{\partial z}, \frac{\partial P}{\partial z} - \frac{\partial R}{\partial x}, \frac{\partial Q}{\partial x} - \frac{\partial P}{\partial y} \right)$$

$$\begin{cases} \frac{\partial R}{\partial y} - \frac{\partial Q}{\partial z} = G_1 = xz \\ \frac{\partial P}{\partial z} - \frac{\partial R}{\partial x} = G_2 = yz \\ \frac{\partial Q}{\partial x} - \frac{\partial P}{\partial y} = G_3 = 1 - z^2 \end{cases}$$

$\boxed{\text{(indeterminado)}}$

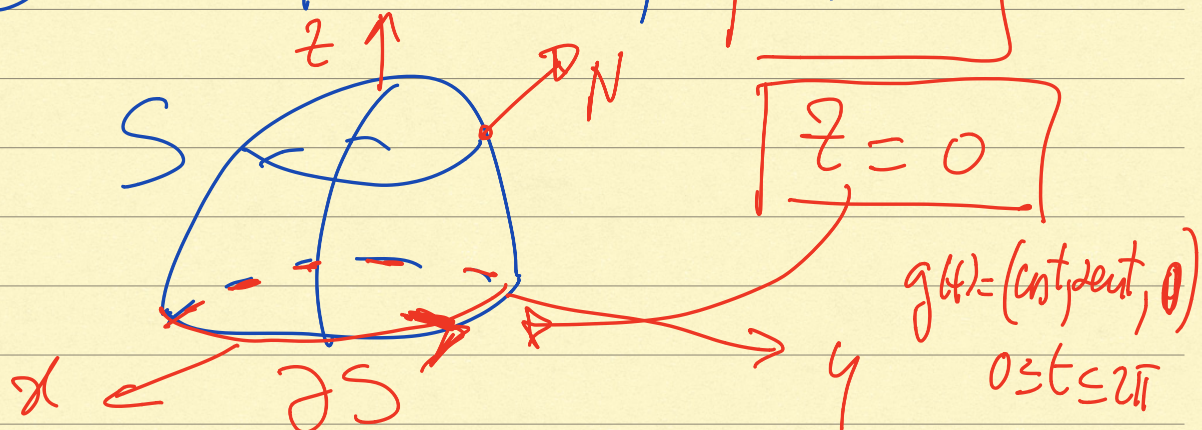
$\Downarrow$

Fazer $P = 0$ ou $Q = 0$ ou $R = 0$

$G = \mathrm{rot}\, F$, $F \equiv$ potencial vectorial de G.

Como se escolhe a componente de $F = (P, Q, R)$ a anular?
Analisar a geometria de S e a definição de trabalho em ∂S.

$S: x^2 + y^2 + z^2 = 1$; $\boxed{z > 0}$



$$\int_{\partial S} F \cdot dg = \int_0^{2\pi} \left(P x' + \underbrace{Q y'} + R \underbrace{\boxed{z'}}_{= 0\ !!!} \right) dt$$

Fazendo $\boxed{P = 0}$,

$$\begin{cases} \dfrac{\partial R}{\partial y} - \dfrac{\partial Q}{\partial z} = xz \\ -\dfrac{\partial R}{\partial x} = yz \longrightarrow R(x,y,z) = -xyz + B(y,z) \\ \dfrac{\partial Q}{\partial x} = 1 - z^2 \longrightarrow Q(x,y,z) = x - xz^2 + A(y,z) \end{cases}$$

b

$A = 0$

$$\Longrightarrow P = 0, \quad Q(x,y,z) = x - xz^2$$

$$R(x,y,z) = -xyz + B(y,z)$$

$$F(x,y,z) = \left(0,\ x - xz^2,\ -xyz + B(y,z)\right)$$

$$\iint_S G\cdot N = \int_{\partial S} F\cdot dg \qquad g(t): \begin{cases} x(t) = \cos t \\ y(t) = \operatorname{sen} t \\ z(t) = 0 \end{cases}$$

$$= \int_0^{2\pi} \left(\underset{=0}{P}\, x' + \underbrace{Q\, y'} + \underbrace{R\, \boxed{z'}}_{=0} \right) dt$$

$$= \int_0^{2\pi} \cos t\,(1-0)\,(-\cos t)\, dt$$

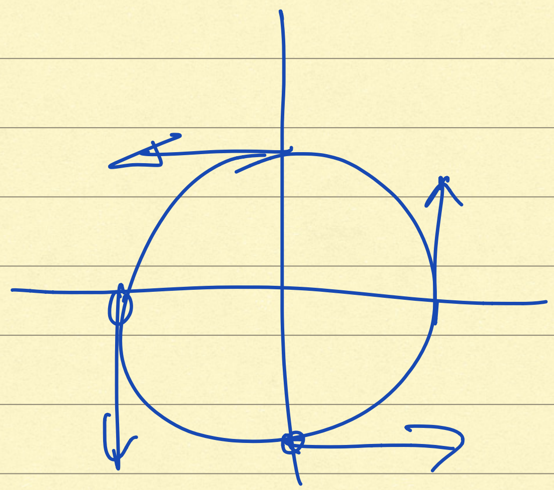
$$= - \int_0^{2\pi} \cos^2 t\, dt$$

$$= - \int_0^{2\pi} \frac{\cos(2t)+1}{2}\, dt \;\ldots$$

Ex. 6 (F-13) (Ralo de banheira)

$\mathbb{R}^2$ (secunda):

$$\left(-\frac{y}{x^2+y^2}, \frac{x}{x^2+y^2} \right)$$



fechado mas não é gradiente.

$(x,y) = (0,0)$

$\mathbb{R}^3$: $H(x,y,z) = \left(\frac{z}{x^2+z^2}+x,\ y,\ -\frac{x}{x^2+z^2}+z\right)$

$$= \underbrace{\left(\frac{z}{x^2+z^2},\ 0,\ -\frac{x}{x^2+z^2}\right)}_{\text{Ralo de banheiro}} + \underbrace{(x, y, z)}_{D\left(\frac{x^2}{2}+\frac{y^2}{2}+\frac{z^2}{2}+C\right)}$$

Fechado mas não é gradiente!

$(x,y,z) \neq (0,y,0)$, $y \in \mathbb{R}$